END SEMESTER EXAMINATION 2017

I[st] SEMESTER

B.Sc. MATHEMATICS HONOURS

CORE COURSE II

Full Marks: 50 Time: 2 hours

(Symbols and notations have their usual meanings)

UNIT-1

Q1. Answer any one question : $1 \times 1 = 1$

a) If a, b, x are real number and $|a+ib| = 1$, prove that $(a+ib)^{ix}$ is purely real.

b) If a,b,c are positive real numbers and abc $=k^3$, prove that $(1+a)(1+b)(1+c) \geq (1+k)^3$.

Q2. Answer any one question : $1 \times 2 = 2$

a) Find the equation each of whose roots is greater by 2 than the roots of the equation

$$x^4+8x^3+x-5=0$$

b) Prove that $\sin^{-1}\sqrt{-1} = n\pi + (-1)^n i\log(\sqrt{2}+1)$ n is any integer.

Q3. Answer any three questions : $3 \times 4 = 12$

a) If the equation $x^4+ax^3+bx^2+cx+d=0$, has three equal roots, then show that each of them is equal to

$$\frac{6c-a.b}{3a^2-8b}.$$

b) If $a_1, a_2, \ldots, a_n; b_1, b_2, \ldots, b_n; c_1, c_2, \ldots, c_n$ be all positive real numbers prove that $(a_1b_1c_1 + a_2b_2c_2 + \cdots + a_nb_nc_n)^2 < (a_1{}^2 + a_2{}^2 + \cdots + a_n{}^2)(b_1{}^2 + b_2{}^2 + \cdots + b_n{}^2)(c_1{}^2 + c_2{}^2 + \cdots + c_n{}^2)$; hence or otherwise show that $(a_1 + a_2 + \cdots + a_n)(\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}) > n^2$, if $a_1, a_2, \ldots, a_n$ are unequal positive real numbers.

c) If $a_1, a_2, \ldots, a_n$ be npositive rational numbers and $s = a_1 + a_2 + \cdots + a_n$, prove that

$$\left(\frac{s-a_1}{n-1}\right)^{a_1}\left(\frac{s-a_2}{n-1}\right)^{a_2}\cdots\left(\frac{s-a_n}{n-1}\right)^{a_n} \leq \left(\frac{s}{n}\right)^s.$$

d) Obtain the equation whose roots are the roots of the equation $4x^3 - 8x^2 - 19x + 26 = 0$ each diminished by 2. Use Descartes' rule of signs to both the equations to find the exact number of positive and negative roots of the given equation.

e) Prove that the equation $(x+1)^4 = a(x^4+1)$ is a reciprocal equation if $a \neq 1$ and solve if when $a = -2$.

UNIT-2

Q4. Answer any one question : $1 \times 1 = 1$

a) Prove that, if $A\Delta B = A\Delta C$, then $B=C$.

b) Calculate gcd(67,15) and express gcd(67,15) as 67u +15v , where u and v are integers

$1 \times 2 = 2$

Q5. Answer any one question :

a) If f be an equivalence relation on a set S, prove that the equivalence classes $cl(a)$ and $cl(b)$ are either equal or disjoint , where $a, b \subset S$.

b) If $f: R \rightarrow R^{+}$ be defined by $f(x) = e^{x}$, $x \in R$, prove that f is invertible and find the inverse function of f.

$3 \times 4 = 12$

Q6. Answer any three questions :

a) State and prove Chinese Remainder Theorem.

b) State the principle of mathematical induction for positive integers and then prove the following inequality by the method of induction

$$(1 + a^2) \geq 1 + na + \frac{n(n-1)}{2}a^2 (a > 0 \text{ and } n \text{ is a positive integer}).$$

c) Show that the relation of congruence modulo m $a \equiv b(\mathrm{mod}\ m)$ in the set of integers Z is an equivalence relation, where $a, b \in Z$.

d) State Peano's Axiom. Show that there are no positive integers strictly between 0 and 1. Show that 23146123 is divisible by 77.

e) Show that $(0,1)$ is uncountable. Show that $\{0,1\} \times \{0,1\} \times \{0,1\} \times \cdots$ is uncountable.

UNIT-3

Q7. Answer any two questions : $2 \times 2 = 4$

a) Define orthogonal matrix. Prove that $detA = \pm 1$ if A is an orthogonal matrix.

b) Find the value of 'k' such that the rank of the matrix $A = \begin{pmatrix} 2 & 6 \\ 7 & k \end{pmatrix}$ is 1.

c) Find the condition that the lines $2x + 3y = 3$ and $x + 2y = 1$ intersect at a point. Find also the point of intersection. $(1 + 1)$

Q8. Answer any one question : $1 \times 4 = 4$

a) Use elementary row operations on A to obtain A^{-1} where $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$.

b) Prove that the necessary and sufficient condition for an non- homogeneous system $AX = B$ to be consistent is rank A = rank $\bar{A}$, where $\bar{A}$ is the augmented matrix of the system.

c) Reduce the following quadratic form to its normal form. Also find the rank and signature of it :

$$2x^2 + 3y^2 + 4z^2 - 4xy + 4yz .$$

UNIT-4

Q9. Answer any two questions : $2 \times 2 = 4$

a) Examine if the set of vectors $\{(1,2,0),(3,-1,1),(4,1,1)\}$ is linearly dependent in $\mathbb{R}^3$.

b) Using Cayley-Hamilton theorem, compute inverse of the matrix A, where $A = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$.

c) If $\lambda\ (\neq 0)$be an eigenvalue of a non-singular matrix A, then prove that $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.

Q10. Answer any two questions : $2 \times 4 = 8$

a) Prove that every square matrix satisfies its own characteristic equation.

b) Diagonalise the matrix $A = \begin{pmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{pmatrix}$.

c) Find the dimension of the subspace S of $\mathbb{R}^4$ defined by

$$S = \{(x,y,z,w) \in \mathbb{R}^4 : x + 2y - z = 0, 2x + y + w = 0\}.$$

d) If T : V → W be a linear transformation where V and W are vector spaces of finite dimensions over a field F. Prove that T is invertible iff the matrix of T is relative to some chosen bases is non-singular.

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