MID SEMESTER EXAMINATION 2017

I[st] SEMESTER

B.Sc. PHYSICS HONOURS

CORE COURSE I

Full Marks: 30 **Time: 1 ½ hours**

Group-A

Answer Question number 1, 2, 3 and 4 taking one from each. Question number 5 and 6 are compulsory. $[2\frac{1}{2}\times 6=15]$

1.

(A) Determine Taylor series about $x=\theta$ for the following integral:

$$\int \frac{e^x-1}{x}\,dx$$

Or

(B) Determine Taylor series for $sin^{-1}x$ about $x=\theta$

2.

(A) Show that the general solution of the second order, linear, nonhomogeneous differential equation $y''+py'+qy=G(x)$ (where p and q are numbers and $G(x)$ is any function continuous on $(-\infty,+\infty)$) can be written as $y_2=c_1 v_1+c_2 v_2+y_1$, where $v_1\ and\ v_2$ are two linearly independent solutions of the corresponding homogeneous equation and $y_1 and\ y_2$ are any two solutions of the nonhomogeneous equation.

Or

(B) Let W be the Wronskian of two solutions $v_1\ and\ v_2$ of the differential equation $y''+py'+qy=0$, where p and q are numbers.

Prove that W satisfies the first order equation $w'+pw=0$ and hence $w(x)=w(0)e^{-px}$.

3.

(A) Give example of a function which is continuous at a point in an interval but not differentiable at that point.(Draw the function and justify your answer)

Show that $Limit\ (x\to 0)\frac{\sin(5x)-\sin(3x)}{x}=2$

Or

(B) Let $f(x)=x+1\ for\ x\le 1\ and\ f(x)=3-ax^2\ for\ x>1$. Determine right hand and left hand limits of $f(x)$ at x=1. Find a such that the function be continuous at x=1. Sketch $f(x)$.

4.

(A) Evaluate $\int_{-\infty}^{+\infty} x^2\delta(x^2+2x-15)dx$

Or

(B) Evaluate $\int_{-\infty}^{+\infty} e^x\delta(x^2-4)dx$

5. If e^{3x} is one solution of the differential equation $y''-6y'+3y=0$, then using Wronskian method determine the second linearly independent solution.

6. The nonhomogeneous Euler equation is given by : $x^2y''-4xy'+6y=g(x)$. If x^3 *and* x^2 be the two linearly independent solutions when $g(x)=0$, then determine a particular solution when $g(x)=\frac{21}{x^4}$. [you can use variation of parameter or any other method.]

Group B

Unit-I (Answer any two questions) $[2\frac{1}{2}\text{X}2=5]$

1. Prove $\nabla^2(\phi\psi)=\phi\nabla^2\psi+2\vec{\nabla}\phi.\vec{\nabla}\psi+\psi\nabla^2\phi$
2. Suppose $\vec{A}$ and $\vec{B}$ are irrotational. Prove that $\vec{A}\,X\,\vec{B}$ is solenoidal
3. Find the constant a so that the following vector are coplanar $3\hat{\imath}-3\hat{\jmath}-\hat{k}$, $-3\hat{\imath}-2\hat{\jmath}+2\hat{k}$, $6\hat{\imath}-a\hat{\jmath}-3\hat{k}$.

Unit-II (Answer any one question) **[10X1=10]**

4. a) Suppose $\vec{A}=x_1\vec{a}+y_1\vec{b}+z_1\vec{c}$, $\vec{B}=x_2\vec{a}+y_2\vec{b}+z_2\vec{c}$, $\vec{C}=x_3\vec{a}+y_3\vec{b}+z_3\vec{c}$. Prove that

$$\vec{A}.\left(\vec{B}X\vec{C}\right)=\begin{vmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{vmatrix}\left(\vec{a}.\left(\vec{b}X\vec{c}\right)\right)$$

[3]

b) Suppose a particle has velocity $\vec{v}$ and acceleration $\vec{a}$ along a space curve C. Prove that the radius of curvature $\rho=\frac{\vec{v}^3}{|\vec{v}X\vec{a}|}$.

[4]

c) Suppose φ(x,y,z) is a scalar invariant with respect to rotation of axes. Prove that $\vec{\nabla}\phi$ is a vector invariant under this transformation.

[3]

5. a) Prove $\nabla^2 f(r)=\frac{d^2f}{dr^2}+\frac{2}{r}\frac{df}{dr}$

[3]

b) Show that $\vec{E}=\frac{\vec{r}}{r^2}$ is irrotational. Find φ such that $\vec{E}=-\vec{\nabla}\phi$ and φ(a)=0 where a>0.4

[4]

c) Prove that $(\vec{v}.\vec{\nabla})\vec{v}=\frac{1}{2}\vec{\nabla}v^2-\vec{v}X(\vec{\nabla}X\vec{v})$.

[3]
