

MID SEMESTER EXAMINATION 2018
FIRST SEMESTER
B.Sc. MATHEMATICS HONOURS
CORE COURSE I

Full Marks: 25

Time: 1 hour

Unit-I

1. Answer any one from the following questions:

2 × 1 = 2

- a) Show that the square of radius of curvature at any point (s, ψ) on the curve $s = 4a \sin \psi$ is $(16a^2 - s^2)$.
- b) Find the slopes of all possible asymptotes of the cubic curve $2x^3 - x^2y + 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0$.

2. Answer any one from the following questions:

4 × 1 = 4

- a) Find the sum of two radii of curvature of the curve $y^2(a - x) = x^2(a + x)$ at the origin.
- b) Find a quadratic polynomial $q(x)$ such that $f(x) = \frac{x^2 - 4}{q(x)}$ has one horizontal asymptote $y = 2$ and two vertical asymptote $x = \pm 3$.

Unit-II

3. Answer any one from the following questions:

2 × 1 = 2

- a) Evaluate $\int_0^{\frac{\pi}{2}} \sin^{\frac{3}{2}} x \cos^3 x \, dx$.
- b) Show that $\int_0^{\frac{\pi}{2}} \cos^2 x \sin 4x \, dx = \frac{1}{3}$.

4. Answer any one from the following questions:

4 × 1 = 4

- a) If $U_n = \int_0^1 x^n \tan^{-1} x \, dx$, prove that for $n > 2$,

$$(n+1)U_n + (n-1)U_{n-2} = \frac{\pi}{2} - \frac{1}{n}.$$

- b) Show that $\int_0^{\frac{\pi}{2}} \cos^n x \, dx = \frac{(n-1)}{n} \cdot \frac{(n-3)}{(n-2)} \cdot \frac{(n-5)}{(n-4)} \dots \dots \dots \frac{1}{2} \cdot \frac{\pi}{2}$, n be any even positive integer greater than 1.

Unit-III

5. Answer any one from the following questions:

2 × 1 = 2

- a) If e and $\acute{e}$ be the eccentricities of a hyperbola and its conjugate, prove that

$$\frac{1}{e^2} - \frac{1}{\acute{e}^2} = 1.$$

- b) A straight line of length $2a$ slides between two rectangular axes OX and OY . Show that the locus of the middle point is $x^2 + y^2 = a^2$.

6. Answer either question no.(a), (b) or (c), (d) :

- a) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 - x - y - z = 0$, $x + y + z = 1$ is a real circle. Hence find the center of the circle. 2
- b) If a right circular cone has three mutually perpendicular generators, Show that its semi-vertical angle is $\tan^{-1} \sqrt{2}$. 2
- c) Find the equations of the circle passing through the points $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$. 1

- d) Show that the angle between the lines given by $x + y + z = 0$, $fyz + gzx + hxy = 0$ is $\frac{\pi}{2}$ if $f + g + h = 0$

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Unit-IV

7. Answer any one from the following questions:

 $3 \times 1 = 3$

- a) Obtain the differential equation of all circles each of which touches the axis of x at the origin.
 b) Solve the Differential equation.
 $y(2xy + e^x)dx - e^x dy = 0$

8. Answer any one from the following questions:

 $4 \times 1 = 4$

- a) Solve: $(x^2y - 2xy^2) dx + (3x^2y - x^3) dy = 0$
 b) Solve: $x^2 dy - \sin 2x dx + 3xy dx = 0$.